Risk statistics examine performance characteristics of a manager or a fund relative to a benchmark (market indicator) that assumes to represent overall movements in the asset class being considered. The mathematical formulas used to calculate these risk statistics are included in the appendix.

Alpha

Alpha represents the historical return from an asset, based on factors unrelated to the underlying factors affecting the market. As such, Alpha is a measure of the return for asset specific (or residual) risk. Alpha is used as a measure of a manager's contribution to performance due to security or sector selection. A positive (negative) Alpha indicates that a portfolio was positively (negatively) rewarded for the residual risk taken for a given level of market exposure. If the market excess return is 2% and the portfolio Beta is 1.1, then the manager would have to have an excess return greater than 2.2% for the manager to have contributed to performance above and beyond the performance of the market.

Formula:

$$\alpha_p = R\left[F\left(\overline{r'_p} - \beta_p \overline{r'_b}\right)\right]^j$$

Where: β_p = Beta of the portfolio

Beta

Beta is a measure of the systematic risk of a security or portfolio. Beta measures the historical sensitivity of portfolio or security excess returns to movements in the excess return of the market index. The value for Beta is expressed as a percentage of the market where the market Beta is 1.00. A security or portfolio with a Beta above the market has volatility greater than the market. If the Beta of a security was 1.3, a 1 percent increase in the market return resulted, on average, in a 1.3 percent increase in the security's return. A security or portfolio with Beta below the market has lower volatility than the market and the return on the security will move less than the market return. If the Beta of the security was .9, a 1 percent decrease in the market resulted in only a .9 percent decrease in the security's return.

Formula:

$$\beta_p = \frac{\sum\left[\left(r'_p - \overline{r'_p}\right)\left(r'_b - \overline{r'_b}\right)\right]}{\sum\left(r'_b - \overline{r'_b}\right)^2}$$

Correlation

Correlation measures the degree to which two variables are associated. Correlation is a commonly used tool for constructing a well-diversified portfolio. Traditionally, equities and fixed-income asset returns have not moved closely together. The asset returns are not strongly correlated. A balanced fund with equities and fixed-income assets represents a diversified portfolio that attempts to take advantage of the low Correlation between the two asset classes. The value for Correlation ranges from +1.0 to -1.0. A positive Correlation means that the two variables move, to a degree, in the same manner or direction, and a negative Correlation means that the variables move, to a degree, in the opposite manner or direction. A Correlation of +1.0 (-1.0) means the two variables move in exactly the same (opposite) direction.

Formula:

$$Corr_p = \frac{\sum\left[\left(r_p - \overline{r_p}\right)\left(r_b - \overline{r_b}\right)\right]}{\sqrt{\sum\left(r_p - \overline{r_p}\right)^2}\sqrt{\sum\left(r_b - \overline{r_b}\right)^2}}$$

Cumulative Relative Returns

The cumulative relative return for a portfolio or asset measures the cumulative return relative to a specified benchmark cumulative return. A period where the portfolio underperformed the benchmark would cause the return ratio for that period to be below zero. However, over the length of the entire time period, the portfolio may have a cumulative return above the benchmark. The relative return value is a ratio where values above zero represent total performance over the period above the benchmark and where values below zero represent total performance over the period below the benchmark.

Formula:

$$cumrelret_p = R\left(\prod \frac{f_p}{f_b}\right)$$

Cumulative Returns

The cumulative return for a portfolio or an asset is the cumulative compound return over the full length of a specified time period. The percentage measure of this return is not annualized and as such represents the actual total return of the portfolio or asset over the period. By annualizing the percentage figure, one can calculate the average annual return of the portfolio or asset over the period.

Formula:

$$cumr_p = R\left(\prod f_p\right)$$

Down Market Capture

Down Market Capture is determined by the index which has a Down-Capture ratio of 100% when the index is performing negatively, if a manager captures less than 100% of the declining market it is said to be "defensive". This statistic is not annualized.

Formula:

$$DownMarket_p = \frac{\left(\prod f_p\right)-1}{\left(\prod f_b\right)-1} \times 100$$

Where $f_b < 0$

Downside Risk

Downside Risk differentiates between “good” risk (upside volatility) and “bad” risk (downside volatility). Whereas standard deviation treats both upside and downside risk the same, downside risk measures only the standard deviation of returns that are below the target. Returns above the target are assigned a deviation of zero. Both the frequency and magnitude of underperformance affect the amount of downside risk.

Formula:

$$DR_p = \sqrt{\frac{\sum\left(r_b - r_p, if\ r_b \rangle r_p, else\ 0\right)^2}{n}} \times \sqrt{j}$$

Excess Correlation

The correlation of one portfolio's excess return to another portfolio's excess return. Excess return is the return minus a benchmark. For instance Excess Correlation could measure the correlation of Manager A's return in excess of a benchmark with Manager B's return in excess of the same benchmark. In PEP the "risk-free-rate" field is the market indicator, which is used to calculate the excess return. The "benchmark" field is the manager whose excess returns will be used as the basis of the correlation. Excess Correlation is used to indicate whether different managers outperform a market index at the same time.

Formula:

$$ExCorr_p = \frac{\sum\left[\left(r'_p - \overline{r'_p}\right)\left(r'_b - \overline{r'_b}\right)\right]}{\sqrt{\sum\left(r'_p - \overline{r'_p}\right)^2}\sqrt{\sum\left(r'_b - \overline{r'_b}\right)^2}}$$

$$r' = (return) - (risk\ \ free\ \ rate)$$

Excess Return

A portfolio return minus the benchmark.

Formula:

$$Excess\ r_p a = R_p - R_b$$

Excess Return Ratio

The Excess Return Ratio is a measure of risk-adjusted relative return. This ratio captures the amount of active management performance (value added relative to an index) per unit of active management risk (tracking error against the index). It is calculated by dividing the manager's annualized cumulative excess return relative to the index by the standard deviation of the individual quarterly excess returns. The Excess Return Ratio can be interpreted as the manager's active risk/reward tradeoff for diverging from the index when the index is mandated to be the "riskless" market position.

Formula:

$$ERR_p = \frac{rel\ ret_p a}{TE_p}$$

Information Ratio

The Information Ratio is a risk statistic that measures the excess return per unit of residual "non-market" risk in a portfolio. The ratio is equal to the Alpha divided by the Residual Risk. Because the Information Ratio represents a residual-risk adjusted measure of the excess returns of a portfolio, the resulting value can be looked at as the excess return per unit of risk that is due solely to the specific risks associated with the securities in the portfolio and by definition could be diversified away.

Formula:

$$IR_p = \frac{\alpha_p}{RR_p}$$

Net Asset Growth

Net Asset Growth measures the net flow to a manager by removing the market impact from reported asset growth. The calculation is based upon the manager's quarterly beginning and ending tax-exempt assets under management and the manager's quarterly return performance. The performance results are removed from the manager's ending assets, then this figure is subtracted from the beginning assets to arrive at the new asset growth. When there are periods with returns but no assets measurements, an iterative process is used to determine the periodic net asset growth for the interleaving periods.

R-Squared(R^2)

R-Squared is a statistical measure that indicates the extent to which the variability of a security or portfolio's returns is explained by the variability of the market. The value will be between 0 and 1. The higher the number, the greater the extent to which portfolio returns are related to the market return. An R-Squared value of .75 indicates that 75% of the fluctuation in a portfolio's return is explained by market action. An R-Squared of 1.0 indicates that portfolio returns are entirely related to the market and are not influenced by other factors. An R-Squared of 0 indicates that no relationship exists between the portfolio's returns and the market return.

Note: That R-Squared measures the strength and not the positive or negative direction of the relationship between assets and the market.

Formula:

$$R^2 = \frac{\left(\sum \left[\left(r'_p - \overline{r'_p} \right) \left(r'_b - \overline{r'_b} \right) \right] \right)^2}{\sum \left(r'_p - \overline{r'_p} \right)^2 \sum \left(r'_b - \overline{r'_b} \right)^2}$$

Real Returns

A portfolio's return minus the risk-free-rate.

Formula:

$$r'_p a = R_p - R_f$$

Relative Returns

The relative return for a portfolio or asset measures the return relative to a specified benchmark return. The relative return value is a ratio where values above zero represent a period where the portfolio outperformed the benchmark and where values below zero represent a period where the portfolio underperformed the benchmark.

Formula:

$$relret_p a = R\left(\prod \left(\frac{f_p}{f_b} \right)^{j/n} \right)$$

Where n > a year. If n < a year, there is no need to annualize the number.

In this case ignore the j/n part of the equation.

Relative Standard Deviation

Relative Standard Deviation is a simple measure of a manager's risk (volatility) relative to a benchmark. It is calculated by dividing the manager's standard deviation of returns by the benchmark's standard deviation of returns. A relative standard deviation of 1.20, for example, means the manager has exhibited 20% more risk than the benchmark over that time period. A ratio of .80 would imply 20% less risk. This ratio is especially useful when analyzing the risk of investment grade fixed income products where actual historical durations are not available. By using this relative risk measure over rolling time periods one can illustrate the "implied" historical duration patterns of the portfolio versus the benchmark.

Formula:

$$rel\sigma_p = \frac{\sigma_p}{\sigma_b}$$

Residual Risk

Residual risk is the unsystematic, firm-specific, or diversifiable risk of a security or portfolio. It is the portion of the total risk of a security or portfolio that is unique to the security or portfolio itself and is not related to the overall market. The residual risk in a portfolio can be decreased by including assets that do not have similar unique risk. For example, a company that relies heavily on oil would have the unique risk associated with a sudden cut in the supply of oil. A company that supplies oil would benefit from a cut in another company's supply of oil. A combination of the two assets helps to cancel out the unique risk of the supply of oil. The level of residual risk in a portfolio is a reflection of the "bets" which the manager places in a particular asset class or sector. Diversification of a portfolio can reduce or eliminate the residual risk of a portfolio.

Formula:

$$RR = \sqrt{\left(1 - R^2\right) \times \frac{\sum\left(r_p' - \overline{r_p'}\right)^2}{(n-2)} \times j}$$

Returns

A standard measure of performance that includes both capital appreciation or depreciation as well as realized gains and losses and income.

Formula:

$$r_p a = R\left[\prod\left(f_p\right)^{j/n}\right]$$

Where n > a year. If n < a year, there is no need to annualize the number.

In this case ignore the j/n part of the equation.

Sharpe Ratio

Sharpe Ratio is a measure of the risk-adjusted return of a portfolio. The ratio represents the return gained per unit of risk taken. The risk of the portfolio is the Standard Deviation of the portfolio returns. The Sharpe ratio can be used to compare the performance of managers. Two managers with the same excess return for a period but different levels of risk will have Sharpe ratios that reflect the difference in the level of risk. The performance of the manager with the lower Sharpe ratio would be interpreted as exhibiting comparatively more risk for the desired return compared to the other manager. If the two managers had the same level of risk but different levels of excess return, the manager with the higher Sharpe ratio would be preferable because the manager achieved higher return with the same level of risk as the other manager. The Sharpe ratio is most helpful when comparing managers with both different returns and different levels of risk. In this case, the Sharpe ratio provides a per-unit measure of the two managers that enables a comparison. The Sharpe Ratio is a risk statistic that measures the excess return per unit of Total Risk taken in a portfolio. The excess return is the total excess return without adjustment for risk. The ratio is equal to the excess return divided by the Standard Deviation of the portfolio.

Formula:

$$SR_p = \frac{\left(R_p - R_f\right)}{\sigma_p}$$

Sortino Ratio

Measuring excess return (not relative return) over a benchmark (e.g., a market index or static threshold, such as 8%) divided by downside risk, the Sortino ratio is intuitively appealing, yet potentially dangerous in its application. The natural appeal is that it identifies value-added per unit of truly bad risk. The danger of interpretation, however, lies in the two areas: (1) the statistical significance of the denominator, and (2) its reliance on the persistence of skewness in return distributions.

As the Sortino ratio's denominator, downside risk needs lots of observations to be meaningful, just like its close relative, standard deviation. While any observation is valid for measuring standard deviation, downside risk reflects only below-target observations. Particularly in a bull market for a Sortino ratio using a low return hurdle, a potentially small number of such observations can make the statistic's significance weak. For example, in the 14 quarters ended 6/30/98, the S&P 500 exhibited no downside risk against a static 8% benchmark; consequently, the Sortino ratio was infinite.

Even if we have a lot of downside observations, downside risk (as opposed to standard deviation) relies on the historical skewness of risk, if any exists. If a portfolio's return distribution has a "fat tail" on the left side of the mean (i.e., a relatively large range of negative variance, vs. that of its positive variance), downside risk can convey potentially important information indicating that much of the portfolio's risk is "bad" risk. Going forward, though, one must assess whether this skewness will persist. Such skewness, however, can easily change especially as we enter a potentially new phase of the market (i.e., bear vs. bull market, or inflationary vs. disinflationary economy). The skewness could change sides. Then, decisions relying on skewness as a measure are going to be worse than relying on standard deviation, which doesn't discriminate between "good" and "bad" risk.

Notwithstanding the above caveats, downside risk and the Sortino ratio, as a downside risk-adjusted measure of value-added, both can offer a helpful supplemental view of performance. If they are indicating a conclusion about a manager that is much different from that using standard deviation and excess return ratio or Sharpe ratio (i.e., when compared against peer performance), then one should understand why.

Formula:

$$Sortino_p = \frac{\left(R_p - R_b\right)}{DR_p}$$

Standard Deviation

Standard Deviation is a statistical measure of portfolio risk. Standard Deviation is equal to the square root of the Variance. It reflects the average deviation of the observations from their sample mean. In the case of portfolio performance, the Standard Deviation describes the average deviation of the portfolio returns from the mean portfolio return over a certain period of time. Standard Deviation measures how wide this range of returns typically is. The wider the typical range of returns, the higher the Standard Deviation of returns, and the higher the portfolio risk. If returns are normally distributed (i.e., has a bell shaped curve distribution), then approximately 2/3 of the returns would occur within plus or minus one Standard Deviation from the sample mean.

Formula:

$$\sigma_p = R\left(\sqrt{\frac{\sum\left(f_p - \overline{f_p}\right)^2}{(n-1)}(j)}\right)$$

Total Risk

Total risk is a measure of the total volatility of the excess returns of an asset or portfolio. The total risk is comprised of two measures of risk: market (non-diversifiable or systematic) risk and residual (diversifiable or firm-specific) risk.

Formula:

$$TR_p = \sqrt{\frac{\sum\left[\left(r'_p - \overline{r'_p}\right)\right]^2}{(n-1)} \times j}$$

Tracking Error

Tracking Error is simply the standard deviation of a portfolio's relative returns (relative to some benchmark). Whereas the standard risk measure of standard deviation measures the absolute return volatility, tracking error measures the volatility of the return differences between the portfolio and the benchmark over time. A portfolio that is actively managed in an aggressive manner would have a large amount of tracking error versus its index, whereas a portfolio that is more constrained to look like its index (an index fund being the extreme) would have smaller amounts of tracking error.

Formula:

$$TE_p = R\left(\sqrt{\frac{\sum\left(f_r - \overline{f_r}\right)^2}{(n-1)}(j)}\right)$$

Where: $f_r = \frac{f_p}{f_b}$

Treynor Ratio

The Treynor Ratio is a risk statistic that measures the excess return per unit of systematic "market" risk taken in a portfolio. The excess return is the total excess return without adjustment for risk. The ratio is equal to the excess return of the portfolio divided by the Beta of the portfolio.

Formula:

$$TR_p = \frac{\left(R_p - R_f\right)}{\beta_p}$$

Up Market Capture Ratio

Up Market Capture is determined by the index which has an Up-Capture ratio of 100% when the index is performing positively, if a manager captures more than 100% of the rising market it is said to be "offensive".

Formula:

$$UpMarket_p = \frac{\left(\prod f_p\right) - 1}{\left(\prod f_b\right) - 1} \times 100$$

Formula Codes

r_p = period return of the portfolio

r_b = period return of the benchmark

r' = real (excess) period return

R = Total Return

R_p = Total Return portfolio

R_b = Total Return benchmark

R_f = Total Return risk-free-rate

f_p = period return of the portfolio in factor format

f_b = period return of the benchmark in factor format

f' = real (excess) return in factor format

F = Total Return in Factor format

n = total number of observations

j = number of observations in a year

If n < year then $j = n$

Analytical Solutions Group

Phone: 415.291.4191 **E-mail: pep@callan.com**
101 California Street, Suite 3500 San Francisco, CA 94111 Fax: 415.291.4018

CALLAN ASSOCIATES INC